DAY TEN

Unit Test 1
(Mechanics)

1 Taking into account the significant figures, what should be the value of $9.99 + 0.0099$?

(a) 9.9999 (b) 10.00 (c) 10.0 (d) 10

2 The maximum error in the measurement of mass and length of the cube are 3% and 2%, respectively. The maximum error in the measurement of density will be

(a) 5% (b) 6% (c) 7% (d) 9%

3 If C and L denote the capacitance and inductance, then the dimensional formula for C-L is same as that for

(a) frequency (b) time period
(c) $(\text{frequency})^2$ (d) $(\text{time period})^2$

4 The dimensions of $(\text{velocity})^2 \div$ radius are the same as that of

(a) Planck's constant (b) Gravitational constant
(c) Dielectric constant (d) None of these

5 A soap bubble oscillates with time period T, which in turn depends on the pressure (p), density (ρ) and surface tension (σ). Which of the following correctly represents the expression for T^2?

(a) $\frac{\rho\sigma^2}{p^3}$ (b) $\frac{\rho p^3}{\sigma}$
(c) $\frac{p^3\sigma}{\rho}$ (d) $\frac{\rho}{p^3\sigma}$

6 An automobile travels on a straight road for 40 km at 30 km h^{-1}. It then continues in the same direction for another 40 km at 60 km h^{-1}. What is the average velocity of the car during its 80 km trip?

(a) 30 km h^{-1} (b) 50 km h^{-1}
(c) 40 km h^{-1} (d) 60 km h^{-1}

7 A particle had a speed of 18 ms^{-1}. After 2.4 s, its speed was 30 ms^{-1} in the opposite direction. What were the magnitude and direction of the average acceleration of the particle during this 2.4 s interval?

(a) 10 ms^{-2} (b) 15 ms^{-2} (c) 20 ms^{-2} (d) 25 ms^{-2}

8 A rock is dropped from a 100 m high cliff. How long does it take to fall first 50 m and the second 50 m?

(a) 2 s, 3 s (b) 1.5 s, 2.5 s
(c) 1.2 s, 3.2 s (d) 3.2 s , 1.3 s

9 Two bodies of masses M_1 and M_2 are dropped from heights H_1 and H_2, respectively. They reach the ground after time T_1 and T_2, respectively. Which of the following relation is correct?

(a) $\frac{T_1}{T_2} = \left[\frac{H_1}{H_2}\right]^{1/2}$ (b) $\frac{T_1}{T_2} = \frac{H_1}{H_2}$
(c) $\frac{T_1}{T_2} = \left[\frac{M_1}{M_2}\right]^{1/2}$ (d) $\frac{T_1}{T_2} = \frac{M_1}{M_2}$

10 As a rocket is accelerating vertically upwards at 9.8 ms^{-2} near the earth's surface, it releases a projectile with zero speed relative to rocket. Immediately after release, the acceleration (in ms^{-2}) of the projectile is [take, $g = 9.8$ ms^{-2}]

(a) zero (b) 9.8 ms^{-2}, up
(c) 9.8 ms^{-2}, down (d) 19.6 ms^{-2}, down

11 A bullet is fired in a horizontal direction with a muzzle velocity of 300 ms^{-1}. In the absence of air resistance, how far will it drop in travelling a horizontal distance of 150 m? [take, $g = 10$ ms^{-2}]

(a) 1.25 cm (b) 12.5 cm (c) 1.25 m (d) 1.25 mm

12 A fixed mortar fires a bomb at an angle of 53° above the horizontal with a muzzle velocity of 80 ms^{-1}. A tank is advancing directly towards the mortar on level ground at a constant speed of 5 ms^{-1}. The initial separation (at the instant mortar is fired) between the mortar and tank, so that the tank would be hit is [take, $g = 10\ ms^{-2}$]

(a) 678.4 m (b) 614.4 m
(c) 64 m (d) None of these

13 A vector **a** is turned without a change in its length through a small angle $d\theta$. The value of $|\Delta \mathbf{a}|$ and Δa are respectively

(a) $0, a\,d\theta$ (b) $a\,d\theta, 0$
(c) 0, 0 (d) None of these

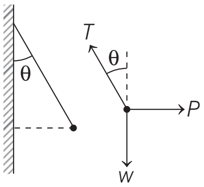
14 The vectors from origin to the points A and B are $\mathbf{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\mathbf{B} = 2\hat{i} + \hat{j} - 2\hat{k}$, respectively. The area of the ΔOAB be

(a) $\frac{5}{2}\sqrt{17}$ sq units (b) $\frac{2}{5}\sqrt{17}$ sq units
(c) $\frac{3}{5}\sqrt{17}$ sq units (d) $\frac{5}{3}\sqrt{17}$ sq units

15 The sum of the magnitudes of the two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitude of the forces?

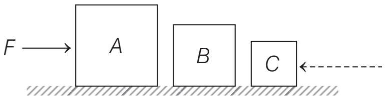
(a) 12, 5 (b) 14, 4 (c) 5, 13 (d) 10, 8

16 A metal sphere is hung by a string fixed to a wall. The sphere is pushed away from the wall by a stick. The forces acting on the sphere are shown in the second diagram. Which of the following statements is wrong?



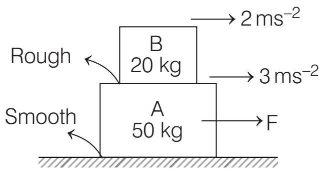
(a) $P = w\tan\theta$ (b) $\mathbf{T} + \mathbf{P} + \mathbf{w} = 0$
(c) $T^2 = P^2 + w^2$ (d) $T = P + w$

17 Three blocks A, B and C of masses 5 kg, 3 kg and 2 kg respectively are placed on a horizontal surface. The coefficient of friction between C and surface is 0.2 while between A and surface is zero and between B and surface is zero. If a force $F = 10$ N is first applied on A as shown and then in second case on C (shown dotted), then the ratio of normal contact force between B and C in first with respect to the second case is [Take, $g = 10\ ms^{-2}$]



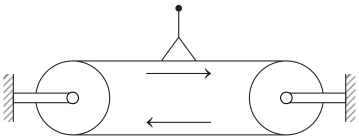
(a) $\frac{13}{12}$ (b) $\frac{12}{13}$ (c) $\frac{1}{2}$ (d) $\frac{2}{1}$

18 A 20 kg block is placed on top of a 50 kg block as shown. A horizontal force F acting on A causes an acceleration of 3 ms^{-2} to A and 2 ms^{-2} to B as shown. For this situation mark out the correct statement.



(a) The friction force between A and B is 40 N
(b) The net force acting on A is 150 N
(c) The value of F is 190 N
(d) All of the above

19 The figure below shows a man standing stationary w.r.t. a horizontal conveyor belt which is accelerating at 1 ms^{-2}. What is the net force on the man in this situation?



Take mass of the person to be as 70 kg. If the maximum acceleration of the belt, for which the man remains stationary w.r.t. the belt, is 3 ms^{-2}, then the coefficient of static friction between the man's shoes and the belt would be

(a) 70 N, 0.2 (b) 70 N, 0.3
(c) 700 N, 0.1 (d) 700 N, 0.3

20 A parachutist is in free fall before opening her parachute. The net force on her has a magnitude F and is directed downwards. This net force is somewhat less than, her weight w because of air resistance. Then, she opens her parachute. At the instant after her parachute fully inflates, the net force on her would be

(a) greater than F and still directed downwards
(b) less than F and still directed downwards
(c) zero
(d) directed upwards but could be more or less than F

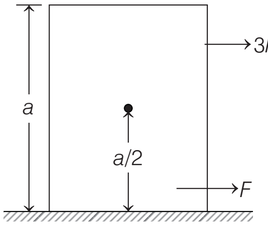
21 The drive shaft of an automobile rotates at 3600 rpm and transmits 80 HP up from the engine to the rear wheels. The torque developed by the engine is

(a) 16.58 N-m (b) 0.022 N-m
(c) 158.31 N-m (d) 141.6 N-m

22 A disk starts rotating from rest about its axis with an angular acceleration equal to $\alpha = 10\ rads^{-2}$, where t is time in seconds. At $t = 0$, the disk is at rest. The time taken by the disk to make its, first complete revolution is

(a) $\left[\frac{6\pi}{5}\right]^{1/3}$ (b) $\left[\frac{3\pi}{10}\right]^{1/3}$ (c) $\left[\frac{2\pi}{5}\right]^{1/2}$ (d) $\left[\frac{6\pi}{13}\right]^{1/3}$

23 A rectangular block of mass M and height a is resting on a smooth level surface. A force F is applied to one corner as shown in the figure. At what point should a parallel force $3F$ be applied in order that the block undergoes pure translational motion? Assume, the normal contact force between the block and surface, passes through the centre of gravity of the block.



(a) $\frac{a}{3}$, vertically above centre of gravity
(b) $\frac{a}{6}$, vertically above centre of gravity
(c) No such point exists
(d) It is not possible

24 A helicopter takes off along the vertical with 3 ms^{-2} with zero initial velocity. In a certain time t, the pilot switches off the engine. The sound dies away at the point of take off in 30 s. When engine is switched off, velocity of the helicopter is

(a) 80 ms^{-1} (b) 30 ms^{-1} (c) 25 ms^{-1} (d) 100 ms^{-1}

25 To maintain a rotor at an uniform angular speed of 200 $rads^{-1}$, an engine needs to transmit a torque of 180 N-m. What is the power required by the engine? (Assume efficiency of the engine to be 80%)

(a) 36 kW (b) 18 kW (c) 45 kW (d) 54 kW

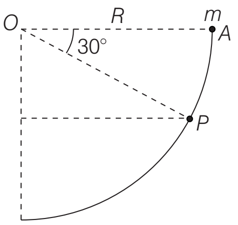
26 When a ball is whirled in a circle and the string supporting the ball is released, the ball flies off tangentially. This is due to

(a) the action of centrifugal force
(b) inertia for linear motion
(c) centripetal force
(d) some unknown cause

27 When a particle is moving in a vertical circle,

(a) its radial and tangential acceleration both are constant
(b) its radial and tangential acceleration both are varying
(c) its radial acceleration is constant but tangential acceleration is varying
(d) its radial acceleration is varying but tangential acceleration is constant

28 A particle of mass m slides on a quarter part of a smooth sphere of radius R as shown in the figure. It is released from rest at A, the normal contact force exerted by surface on the particle, when it reaches P is



(a) $\frac{mg}{2}$ (b) $\frac{3mg}{2}$
(c) $mg \times \frac{\sqrt{3}}{2} + mg$ (d) $\frac{mg\sqrt{3}}{2}$

29 If an object weighs 270 N at the earth's surface, what will be the weight of the object at an altitude equal to twice the radius of the earth?

(a) 270 N (b) 90 N
(c) 30 N (d) 60 N

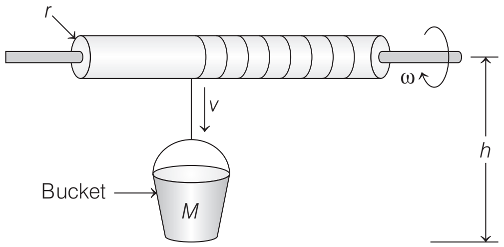
30 At its aphelion, the planet mercury is 6.99×10^{10} m from the sun, and at its perihelion it is 4.6×10^{10} m from the sun. If its orbital speed at aphelion is 3.88×10^4 ms^{-1}, then its perihelion orbital speed would be

(a) 3.88×10^4 ms^{-1} (b) 5.90×10^4 ms^{-1}
(c) 5.00×10^4 ms^{-1} (d) 5.5×10^4 ms^{-1}

31 If R is the radius of the orbit of a geosynchronous satellite and another satellite is orbiting around the earth in a circular orbit of radius $\frac{R}{2}$, then its time period would be

(a) $6\sqrt{2}$ h (b) 6 h
(c) 12 h (d) Cannot be determined

32 A cylinder of mass M and radius r is mounted on a frictionless axle over a well. A rope of negligible mass is wrapped around the cylinder and a bucket of mass m is suspended from the rope. The linear acceleration of the bucket will be



(a) $\frac{mg}{M + 2m}$ (b) $\frac{2mg}{m + 2M}$
(c) $\frac{mg}{2M + m}$ (d) $\frac{2mg}{M + 2m}$

33 A merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round of afterwards is

(a) 2ω (b) ω (c) $\omega/2$ d) 0

34 A canon ball is fired with a velocity of 200 ms^{-1} at an angle of 60° with the horizontal. At the highest point, it explodes into three equal fragments. One goes vertically upwards with a velocity of 100 ms^{-1} and other goes vertically downwards with 100 ms^{-1}. The third one moves with a velocity of

(a) 100 ms^{-1}, horizontally
(b) 300 ms^{-1}, horizontally
(c) 200 ms^{-1}, at 60° with horizontal
(d) 300 ms^{-1}, at 60° with horizontal

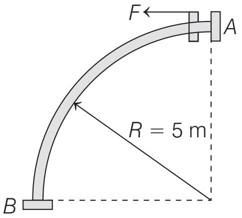
35 A rocket of initial mass (including fuel) 15000 kg ejects mass at a constant rate of 25 kgs^{-1} with a constant relative speed of 15 kms^{-1}. The acceleration of the rocket, 5 min after the blast is [Neglect gravity effect]

(a) 40 ms^{-2} (b) 50 ms^{-2}
(c) 60 ms^{-2} (d) 45 ms^{-2}

36 An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 ms^{-1}. Frictional force of 2000 N is opposing its motion. The minimum power delivered by the motor to the elevator is [take, $g = 10\,ms^{-2}$]

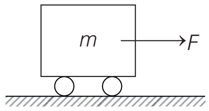
(a) 36 kW (b) 4 kW
(c) 40 kW (d) – 40 kW

37 A bead of mass 1/2 kg starts from rest from a point A to B move in a vertical plane along a smooth fixed quarter ring of radius 5 m, under the action of a constant horizontal force $F = 5$ N as shown. The speed of the bead as it reaches the point B is
[Take $g = 10\,ms^{-2}$]



(a) 14.14 ms^{-1} (b) 7.07 ms^{-1} (c) 5 ms^{-1} (d) 25 ms^{-1}

38 A car (treat it as particle) of mass m is accelerating on a level smooth road under the action of a single force F. The power delivered to the car is constant and equal to P. If the velocity of the car at an instant is v, then after travelling how much distance, it becomes double?



(a) $\frac{7mv^3}{3P}$ (b) $\frac{4mv^3}{3P}$ (c) $\frac{mv^3}{P}$ (d) $\frac{18mv^3}{7P}$

Direction (Q. Nos. 39-40) *Each of these questions contains two statements Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

39 By considering the earth to be non-spherical

Statement I As, one moves from equator to the pole of the earth, the value of accelaration due to gravity increases.

Statement II If the earth stops rotating about its own axis, the value of accelaration due to gravity will be same at pole and at equator.

40 **Statement I** Total torque on a system is independent of the origin if the total external force is zero.

Statement II Torque due to a couple is independent of the origin.

ANSWERS

1. (d)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)
11. (c)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (d)
21. (c)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (b)
31. (a)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (c)	**40.** (a)

Hints and Explanations

1 $9.99 + 0.0099 = 9.9999 = 10$

2 Here, $\frac{\Delta M}{M} = 3\% = \pm\ 0.03$

and $\frac{\Delta l}{l} = 2\% = \pm\ 0.02$

Hence, $\frac{\Delta V}{V} = \frac{3\Delta l}{l} = \pm\ 0.06$

Now, $\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V}$

$\left[\because \text{Density} = \frac{\text{Mass}}{\text{Volume}}\right]$

$= \pm\ 0.09 = 9\%$

3 Time period of C-L oscillations is given by $2\pi\sqrt{LC}$.

Hence, $[LC] = [\text{time period}]^2$

4 Dimensional formula of $(\text{velocity})^2 \div$

radius $= \frac{[M^0LT^{-1}]^2}{[M^0LT^0]} = [M^0LT^{-2}]$

$= [\text{acceleration}]$

Note *In circular motion, centripetal acceleration is* $\frac{V^2}{R}$.

5 Here, $T^2 = p^a \rho^b \sigma^c$ ...(i)

Putting the dimensions of the quantities in RHS, we get

$= [ML^{-1}T^{-2}]^a [ML^{-3}]^b\ [MT^{-2}]^c$

$= [M^{a+b+c} L^{-a-3b} T^{-2a-2c}]$

Hence, $a + b + c = 0$ ...(ii)

$-a - 3b = 0$ ...(iii)

and $-2a - 2c = 2$ ...(iv)

On solving, Eqs. (ii) (iii) and (iv), we get

$a = -3b, b = 1$ and $c = 2$

So, after putting the values of a, b and c in Eq. (i), we get

$$T^2 = \frac{\rho\sigma^2}{p^3}$$

6 Average velocity $(v_{av}) = \frac{\Delta x}{\Delta t}$

where, Δx is the displacement in a given time interval. For first part of the journey, time taken by the car,

$$\Delta t_1 = \frac{40}{30} = 1.33 \text{ h}$$

For second part of the journey time taken by the car

$$\Delta t_2 = \frac{40}{60} = 0.67 \text{ h}$$

Hence, the total displacement

$$\Delta x = \Delta x_1 + \Delta x_2 = 40 + 40 = 80 \text{ km}$$

and the total time interval

$\Delta t = \Delta t_1 + \Delta t_2 = 1.33 + 0.67 = 2$ h

The average velocity is

$$v_{av} = \frac{80}{2} = 40 \text{ kmh}^{-1}$$

7 The average acceleration in the given interval is

$$a_{av} = \frac{(v_2 - v_1)}{(t_2 - t_1)}$$

Take, $v_1 = 18 \text{ ms}^{-1}, v_2 = -30 \text{ ms}^{-1}$,

$t_1 = 0$ and $t_2 = 2.4$s

$$a_{av} = \frac{-30 - 18}{2.4} = -20 \text{ ms}^{-2}$$

The – ve sign indicates that the acceleration is opposite to the initial direction of motion.

8 Using second equation of motion,

$y = ut + \frac{1}{2}at^2$, at $v_0 = 0$

and solve $y = v_0 t + \frac{1}{2}gt^2$ for t

So, $t = \sqrt{2y/g}$

For $y = 50$ m

So, $t_1 = \sqrt{\frac{2(50)}{9.8}} = 3.2$ s

For $y = 100$ m

$t_2 = \sqrt{\frac{2\ (100)}{9.8}} = 4.5$ s

Hence, the difference is the time taken to fall the second 50 m

$= 4.5 - 3.2 = 1.3$ s

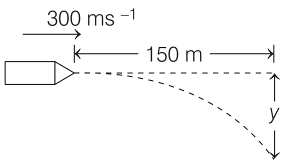
9 Distance of fall is independent of the mass of the bodies

$H_1 = \frac{1}{2}gT_1^2$ and $H_2 = \frac{1}{2}gT_2^2$

Hence, $\frac{T_1}{T_2} = \left[\frac{H_1}{H_2}\right]^{1/2}$

10 As nothing has been mentioned that w.r.t. which frame of reference is to be found, it means we have to compute w.r.t frame of reference of earth. As the object is released, its acceleration w.r.t ground is only due to the influence of gravity of the earth and hence is equal to 9.8 ms^{-2} in the downward direction.

11 Let the bullet, dropped by y metre while covering a horizontal distance of 150 m.



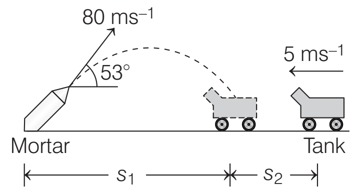
Let t be the time taken by the bullet to cover a horizontal distance of 150 m, then

$150 = 300t \Rightarrow t = \frac{1}{2}$s

$y = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times \frac{1}{4} = 1.25$ m

12 The situation is shown clearly in figure.

Time of flight of bomb is $T = \frac{2u \sin\theta}{g}$



$= \frac{2 \times 80 \times 4}{10 \times 5} = 12.8$s

Distance travelled by the tank in T seconds is

$s_1 = 5T = 5 \times 12.8 = 64$ m

The horizontal distance travelled by bomb in T seconds is

$$s_2 = \frac{u^2 \sin 2\theta}{g} = \frac{80^2 \times 2 \times \frac{3}{5} \times \frac{4}{5}}{10} = 614.4 \text{ m}$$

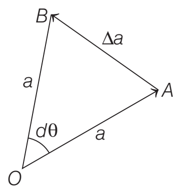
$[\because \sin\ 2\theta = 2\sin\theta \cdot \cos\theta]$

So, required separation

$s = s_1 + s_2 = 678.4$ m

13 From the figure $|\mathbf{OA}| = a$ and $|\mathbf{OB}| = a$.

Also, from the triangle rule



$\mathbf{OB} - \mathbf{OA} = \mathbf{AB} = \Delta\mathbf{a}$

$\Rightarrow |\Delta\mathbf{a}| = AB$

Using, Angle $= \frac{\text{Arc}}{\text{Radius}}$

$\Rightarrow AB = a \cdot d\theta$

So, $|\Delta\mathbf{a}| = a \cdot d\theta$

Δa means a change in magnitude of the vector, i.e.,

$|\mathbf{OB}| - |\mathbf{OA}| \Rightarrow a - a = 0$

So, $\Delta a = 0$

14 Given $\mathbf{OA} = \mathbf{a} = 3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{OB} = \mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$$\therefore \quad (\mathbf{a} \times \mathbf{b}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= (12 - 2)\hat{\mathbf{i}} + (4 + 6)\hat{\mathbf{j}} + (3 + 12)\hat{\mathbf{k}}$$

$$= 10\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 15\hat{\mathbf{k}}$$

$$\Rightarrow \quad |\mathbf{a} \times \mathbf{b}| = \sqrt{10^2 + 10^2 + 15^2} = \sqrt{425} = 5\sqrt{17}$$

$$\text{Area of } \Delta OAB = \frac{1}{2}|\mathbf{a} \times \mathbf{b}| = \frac{5\sqrt{17}}{2} \text{ sq units.}$$

15 Let P be the smaller force and Q be the greater force, then according to the problem

$$P + Q = 18 \quad \ldots \text{(i)}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12 \quad \ldots \text{(ii)}$$

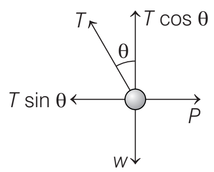
$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90^\circ = \infty$$

$$\therefore \quad P + Q\cos\theta = 0 \quad \ldots \text{(iii)}$$

By solving Eqs. (i), (ii) and (iii), we get

$$P = 5 \text{ and } Q = 13$$

16



As the metal sphere is in equilibrium under the effect of the three forces therefore, $T + P + w = 0$

From the figure, $T\cos\theta = w$ ...(i)

$T\sin\theta = P$ ... (ii)

From Eqs. (i) and (ii), we get

$$P = W\tan\theta$$

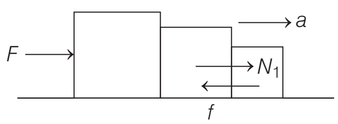
and $$T^2 = P^2 + w^2$$

[as, $\sin^2\theta + \cos^2\theta = 1$]

17 Friction force between C and surface is $f = \mu \times 2g = 0.2 \times 2 \times 10 = 4\text{N}$

***Case* I** $(5 + 3 + 2)a = F - f$

$$\Rightarrow \quad a = \frac{10 - 4}{10} = \frac{6}{10}$$

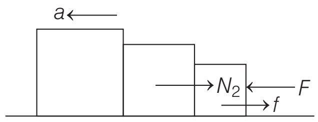


For C, $N_1 - f = 2a$

$$\Rightarrow \quad N_1 = 2 \times \frac{6}{10} + 4 = \frac{26}{5}\text{ N}$$

***Case* II** $F - f = (5 + 3 + 2)a$

$$\Rightarrow \quad a = \frac{10 - 4}{10} = \frac{6}{10}$$

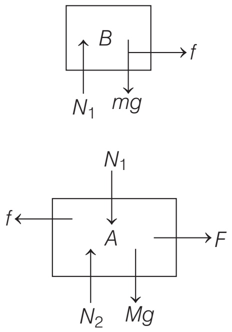


For C, $F - N_2 - f = 2a$

$$\Rightarrow \quad N_2 = 10 - 4 - 2 \times \frac{6}{10} = \frac{24}{5}\text{N}$$

$$\text{Required ratio is, } \frac{N_1}{N_2} = \frac{26}{24} = \frac{13}{12}$$

18 As the acceleration of A and B are different, it means there is a relative motion between A and B. The free body diagram of A and B can be drawn as



For A, $F - f = Ma_A = 50 \times 3 = 150$ N

For B, $f = ma_B = 20 \times 2 = 40$ N

So, $F = 150 + 40 = 190$ N

19 As person remains stationary w.r.t. belt, so acceleration of the person w.r.t. ground is the same as that of belt w.r.t. ground. So, net force acting on the person is, $f = ma = 70 \times 1 = 70$N. Let coefficient of static friction between the man's shoes and belt is μ_S, then

$$a_{\max} = \mu_s g$$

$$\mu_S = \frac{3}{10} = 0.3$$

20 As the parachute inflates fully, the force of air friction increases by a large amount and the parachutist starts decelerating, i.e. net force acting on her is in upward direction but the magnitude of the net force cannot be determined from given information.

21 From $\quad P = \tau\omega$

$$\tau = \frac{P}{\omega}$$

It is given,

$P = 80\text{ HP} = 80 \times 746$ W

$= 59680$ N- ms^{-1}

$$\omega = 3600\text{ rpm} = \frac{3600}{60} \times 2\pi\text{ rad s}^{-1}$$

$$= 120\pi\text{ rad s}^{-1}$$

So, $\tau = 158.31$ N-m

22 As, $\quad \omega = \omega_0 \pm \alpha t$

$$\Rightarrow \quad \omega_0 = 10t$$

$$\text{or} \quad \frac{d\theta}{dt} = 10t$$

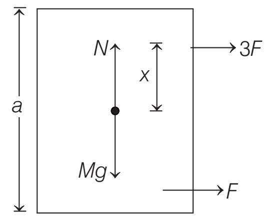
On integrating, (for one complete revolution)

$$\int_0^{2\pi} d\theta = \int_0^T 10t\,dt$$

$$\Rightarrow \quad 2\pi = \frac{10T^2}{2}$$

$$\Rightarrow \quad T = \left(\frac{2\pi}{5}\right)^{\frac{1}{2}}$$

23 The free body diagram of the block can be drawn as shown. As body has to move in a pure translational motion, the torque about the centre of gravity must be zero.



$$3F \times x = F \times \frac{a}{2} \quad \Rightarrow \quad x = \frac{a}{6}$$

24 The altitude of the helicopter when engine is switched off $h = \frac{at_1^2}{2}$. Sound is not heard after $t_2 = t_1 + \frac{at_1^2}{2c}$, where c = speed of sound.

$$at_1^2 + 2ct_1 - 2ct_2 = 0$$

$$\Rightarrow \quad t_1 = \frac{-2c \pm \sqrt{4c^2 + 8cat_2}}{2a}$$

$$t_1 = -\frac{c}{a} + \sqrt{\frac{c^2}{a^2} + \frac{2c}{a}t_2}$$

$$\therefore \; v = at_1 = -c + \sqrt{c^2 + 2cat_2}$$

$$= -320 + \sqrt{(320)^2 + 2 \times 320 \times 3 \times 30}$$

$$= \sqrt{1600 \times (10)^2} - 320$$

$$= 400 - 320 = 80\text{ ms}^{-1}$$

25 Power required for rotor, $P = \tau \cdot \omega$

$= 180 \times 200 = 36$ kW

Power of engine, $P_0 = \frac{P}{0.8} = 45$ kW

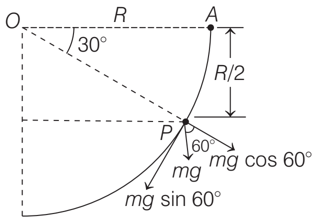
[as efficiency is 80%]

26 This is due to Intertia for linear motion of ball.

27 In a vertical circle, both radial and tangential components of the acceleration change direction at every instant.

28 Apply Work-Energy theorem at A and P,

$$\frac{mv^2}{2} - 0 = mg \times \frac{R}{2}$$



Use dynamical equations at P,

$$N - mg\cos 60° = \frac{mv^2}{R}$$

$$\Rightarrow \quad N = mg + \frac{mg}{2} = \frac{3\,mg}{2}$$

29 Let m be the mass of the object and g is the acceleration due to gravity at the earth's surface, then $mg = 270$ N.

The acceleration due to gravity at an altitude of $2R_e$ is,

$$g' = \frac{GM}{(R_e + 2R_e)^2} = \frac{GM}{9R_e^2} = \frac{1}{9} \times g$$

So, required weight

$$= mg' = \frac{mg}{9} = \frac{270}{9} = 30 \text{ N}$$

30 From the conservation of angular momentum,

$$m\, v_A r_A = m v_p r_p$$

$$\Rightarrow v_p = \frac{3.88 \times 10^4 \times 6.99 \times 10^{10}}{4.6 \times 10^{10}}$$

$$= 5.90 \times 10^4 \text{ ms}^{-1}$$

31 From the Kepler's law, $T^2 \propto r^3$

$$\Rightarrow \quad \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow \quad \frac{24}{T_2} = \left(\frac{R}{R/2}\right)^{3/2} = 2^{3/2}$$

$$\Rightarrow \quad T_2 = \frac{24}{2^{3/2}} = \frac{24}{2\sqrt{2}} = 6\sqrt{2}\text{ h}$$

32 Weight of bucket acts downwards while tension T in opposite direction

$$mg - T = ma$$

Also, $\quad \tau = I\,\alpha = rT$

$$\Rightarrow \quad \frac{1}{2} Mr^2\alpha = rT \Rightarrow \frac{1}{2} M(r\alpha) = T$$

or $\quad T = \dfrac{Ma}{2}$

$$\therefore \quad a = \frac{mg - T}{m}$$

$$\Rightarrow \quad a = \frac{mg - \frac{Ma}{2}}{m} = \frac{2mg - Ma}{2m}$$

$$\Rightarrow 2ma + Ma = 2mg$$

$$\Rightarrow \quad a = \frac{2mg}{2m + M}$$

33 As no external torque acts on the system, angular momentum should be conserved.

Hence, $I\omega$ = constant ...(i)

where, I is moment of inertia of the system and ω is angular velocity of the system.

From Eq. (i), we get

$$I_1\omega_1 = I_2\omega_2$$

[where ω_1 and ω_2 are angular velocities before and after jumping]

$$\Rightarrow \quad I\omega = \frac{I}{2} \times \omega_2$$

[as mass reduced to half, hence moment of inertia also reduced to half]

$$\Rightarrow \quad \omega_2 = 2\omega$$

34 At the highest point, velocity before explosion is $v\cos 60°$ along X-axis.

By law of conservation of momentum,

$$(mv\cos 60°)\,\hat{\mathbf{i}} = \frac{m}{3}(100\hat{\mathbf{j}}) + \frac{m}{3}(-100\,\hat{\mathbf{j}}) + \frac{mv'}{3}$$

$$\Rightarrow \quad v' = \frac{3v}{2} = \frac{3 \times 200}{2}\hat{\mathbf{i}}$$

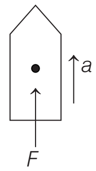
$$= 300\,\hat{\mathbf{i}} \text{ ms}^{-1}$$

or 300 ms^{-1} along X-axis or horizontally.

35 Thrust force acting on the rocket is,

$$F = v_{\text{rel}} \frac{dm}{dt}$$

$$F = 15 \times 1000 \times 25\text{N}$$



Mass of rocket at $t = 5$ min after the blasting starts, is

$$m = 15000 - 25 \times 5 \times 60 = 7500$$

So, $\quad F = ma$

$$\Rightarrow \quad a = \frac{F}{m}$$

$$= \frac{15 \times 25000}{7500}$$

$$= 50 \text{ ms}^{-2}$$

Note *If gravity is not neglected, then* $F - mg = ma$ *check whether acceleration is constant here.*

36 The net downward force on the elevator is,

$$F_1 = mg + f = 18000 + 2000 = 20000 \text{ N}$$

So, the motor has to work against this force.

To move the elevator with a constant speed, the minimum power delivered by the motor to the elevator must be,

$$P = \mathbf{F} \cdot \mathbf{v} = 20000 \times 2 = 40 \text{ kW}$$

37 Applying the work-energy theorem,

$$\frac{1}{2} \times mv^2 - 0 = F \times R + mg \times R$$

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times v^2 = 5 \times 5 + \frac{1}{2} \times 10 \times 5 = 50$$

$$v = \sqrt{200}$$

$$= 14.14 \text{ ms}^{-1}$$

38 As, $P = Fv = mv\dfrac{dv}{ds} \times v$

$$\Rightarrow \quad \int_v^{2v} mv^2 dv = \int_0^s P\,ds$$

$$\Rightarrow \quad s = \frac{7mv^3}{3P}$$

39 As one go from equator to pole of the earth, the value of g increase due to decrease in latitude (λ). Also, the earth is non-spherical, this implies the value of g, at the poles and equitorial point on the earth's surface are unequal due to its different distances from earth's centre.

40 If net force on the system is zero, it can be resolved into two equal and opposite forces which can be considered to form a couple.